Last time: → Algebra of prop. Equiv
→ Disjunctive Normal Form
→ Arguments ← ← ←

Rules of Inference

Name	Form
Modus Ponens	$P \to Q,\ P \ \therefore Q$
Modus Tollens	$P \to Q,\ \neg Q \ \therefore \neg P$
Disjunctive Syllogism	$P \vee Q,\ \neg P \ \therefore Q$ ←
Hypothetical Syllogism	$P \to Q,\ Q \to R \ \therefore P \to R$
Dilemma	$P \vee Q,\ P \to R,\ Q \to R \ \therefore R$
Reductio Ad Absurdum	$(\neg P) \to (Q \wedge (\neg Q)) \ \therefore P$
Conjunctive Addition	$P, Q \ \therefore P \wedge Q$
Conjunctive Simplification	$P \wedge Q \ \therefore P$
Disjunctive Addition	$P \ \therefore P \vee Q$
EQUIVALENT EXCHANGE	$P \equiv Q,\ P \overset{\text{replace}}{\rightsquigarrow} Q$

↳ Ex: $(\neg P) \wedge (\neg Q) \rightsquigarrow \neg(P \vee Q)$

Ex: Verify Disjunctive syllogism is a [valid argument] via truth table

Sol: $P \vee Q,\ \neg P \ \therefore Q \rightsquigarrow [((P \vee Q) \wedge (\neg P)) \to Q]$

P	Q	P ∨ Q	¬P	(P ∨ Q) ∧ (¬P)	((P ∨ Q) ∧ (¬P)) → Q
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

∴ Disj. Syll. is valid ✓

Natural Deduction

(Modified version of Fitch + Lemon Logic System)

We'll use a 2-column format for making proofs.

num	Statements [Formula]	Justification [Rule/Equivalence + line numbers]

Ex: Prove $\neg((\neg P) \wedge (\neg Q)) \therefore P \vee Q$.

Sol:

n	Statements	Justifications	
1	$\neg((\neg P) \wedge (\neg Q))$	Assumption	(Assume)
2	$(\neg(\neg P)) \vee (\neg(\neg Q))$	De Morgan Rule 1	(DM 1)
3	$P \vee (\neg(\neg Q))$	Double Negation 2	(DN 2)
4	$P \vee Q$	DN 3	☑

NB:
- → All assumptions appear on their own line at the beginning of the proof.
- → All lines are numbered successively from 1.
- → Every line comes w/ justification
- → Every non-assumption justification has named rule (or abbreviation) w/ line numbers.
- → Each new statement is the result of one and only one rule application.
- → The last statement is the conclusion!

Ex: Prove $\neg((\neg P) \wedge (\neg Q)) \therefore P \vee Q$ (again.)

Sol:

n	Statement	Justification
1	$\neg((\neg P) \wedge (\neg Q))$	Assumption
2	$\neg(\neg(P \vee Q))$	DM 1
3	$P \vee Q$	DN 2 ☑

Ex: $(\neg P) \rightarrow ((\neg P) \rightarrow (Q \rightarrow P)), \neg P \therefore \neg Q$.

Sol:

n	Statement	Justification
1	$(\neg P) \rightarrow ((\neg P) \rightarrow (Q \rightarrow P))$	Assumed
2	$\neg P$	Assumed
3	$(\neg P) \rightarrow (Q \rightarrow P)$	Modus Ponens 1,2 (MP 1,2)
4	$Q \rightarrow P$	MP 3,2
5	$\neg Q$	Modus Tollens 4,2 (MT 4,2) ☑

$P \rightarrow (Q \rightarrow R), Q \therefore P \rightarrow R$

Ex: $P \rightarrow Q, P \rightarrow R \therefore P \rightarrow (Q \wedge R)$

Sol: ON HOLD. (Remind me!)

View of Coming Attractions:

- → Conditional Proof (method for proving implications)
- → Proof By Contradiction